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1994 J. Phys. A: Math. Gen. 27 L887

(http://iopscience.iop.org/0305-4470/27/23/004)

J. Phys. A: Math. Gen. 27 (1994) L887–L890. Printed in the UK

LETTER TO THE EDITOR

Tricritical phenomena in a $Z(3)$ lattice gauge theory

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Received 30 August 1994

Abstract. A $Z(3)$ gauge model with double plaquette representation of the action on a generalized Bethe lattice of plaquettes is constructed. It is reduced to the spin-1 Blume–Emery–Griffiths (BEG) model. An Ising-type critical line of a second-order phase transition ending in the tricritical point is found.

A continuum limit of a lattice gauge theory may be constructed at the point of a second-order phase transition, since infinite-range correlations allow us to wipe out lattice effects.

At a tricritical or multicritical point the number of relevant couplings is larger than at the vicinity of a second-order critical line. Therefore the existence of such points in the lattice gauge theories opens a possibility for further non-trivial continuum limits.

The purpose of this letter is a search for multicritical points in the strong-coupling region of $Z(3)$ lattice gauge theory.

According to Kogut's formulation of a gauge Potts model on a lattice [1], we constructed the $Z(3)$ gauge model on the generalized Bethe lattice of plaquettes [2] with double plaquette (window) representation of the action. The choice of this mixed action allowed us to connect it with the Hamiltonian of the spin-1 BEG model [3]. Therefore our gauge model can be investigated by applying various results obtained for the BEG model [3–9]. In our previous paper [10] we considered this gauge model on flat triangular and square lattices. Using the exact solution of the BEG model on the honeycomb lattice [6] and the solution obtained by the free-fermion approximation technique on the square lattice [7], we found the lines of the second-order phase transition.

The generalized Bethe lattice is a generalization of a Cayley tree. For our purpose we only consider it with coordination number $q = 2$ (the number of plaquettes coming out from one link). The dual lattice is constructed by joining each nearest plaquettes centres and as a result we get the usual Bethe lattice with coordination number $q = 4$.

Dual lattices for the generalized Bethe ones with coordination numbers higher than 2 are more complicated hierarchical lattices. For example, if we consider the generalized Bethe lattice with $q = 3$ then the dual lattice will be a Husimi lattice [11].

The enlarged lattice gauge actions involving new double plaquette interaction terms were proposed and studied in 3D and 4D by Edgar [12] and Bhanot *et al* [13]. The 2D

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0305-4470/94/230887+04$19.50

version of one of these lattice gauge models with $Z(2)$ gauge symmetry formulated on planar rectangular windows was investigated by Turban [14]. This model with pure gauge action on the square lattice had been reduced to the usual spin-$\frac{1}{2}$ Ising model and the point of a second-order phase transition was found.

The model is considered in terms of the bond variables U_b which take their values in the $Z(3)$, the group of the third roots of unity. Let $U_{p_i} = \prod_{b\in\partial p} U_b$ denote the product of U_b's around an elementary plaquette i.

The gauge invariant action of the model is

$$S_{\mathrm{Gauge}}(\alpha_{2g}, \beta_{2g}, \beta_g) = S_{pp} + S_p \tag{1}$$

where

$$S_{pp} = -\sum_{\langle p_i\, p_j\rangle} \{\alpha_{2g}(\delta_{U_{p_i},1}\delta_{U_{p_j},1} + \delta_{U_{p_i},z}\delta_{U_{p_j},z}) + \beta_{2g}(\delta_{U_{p_i},1}\delta_{U_{p_j},z} + \delta_{U_{p_i},z}\delta_{U_{p_j},1})\}$$

$$S_p = \beta_g \sum_{p_i} \left(\delta_{U_{p_i}1} + \delta_{U_{p_i}z}\right)$$

where U_{p_i} denotes the usual plaquette variable, the product of link gauge fields $U_{x,\mu}$ around an elementary plaquette. The first summation goes over all nearest-neighbour plaquettes and the second one is over all plaquettes of the lattice, $z = \exp\left(\mathrm{i}\frac{2}{3}\pi\right) \in Z(3)$.

After introduction of spin variables S_i in the sites of the dual lattice such that

$$S_i = \delta_{U_{p_i}1} - \delta_{U_{p_i}z} \qquad S_i^2 = \delta_{U_{p_i}1} + \delta_{U_{p_i}z} \tag{2}$$

the action (1) becomes

$$S_{\mathrm{Spin}}(\alpha_{2g}, \beta_{2g}, \beta_g) = -\sum_{\langle ij\rangle} \left\{\tfrac{1}{2}(\alpha_{2g} - \beta_{2g})S_iS_j + \tfrac{1}{2}(\alpha_{2g} + \beta_{2g})S_i^2S_j^2\right\} + \beta_g \sum_i S_i^2 \tag{3}$$

in which we recognize the Hamiltonian multiplied by $1/k_BT$ of the well known BEG model [3].

The corresponding partition function of the model (1) on the generalized Bethe lattice is

$$Z_{\mathrm{Gauge}}(\alpha_{2g}\beta_{2g}\beta_g) = \sum_{\{U\}} \exp[-S_{\mathrm{Gauge}}(\alpha_{2g}\beta_{2g}\beta_g)] \tag{4}$$

where the sum is taken over all possible configurations of the gauge variables $\{U\}$. This partition function can be rewritten in terms of the spin variables S_i defined in the sites of the dual lattice

$$Z_{\mathrm{Gauge}} = 3^N Z_{\mathrm{Spin}}^{\mathrm{Dual}} \tag{5}$$

where

$$Z_{\mathrm{Spin}}^{\mathrm{Dual}} = \sum_{\{S\}} \exp[-S_{\mathrm{Spin}}].$$

A factor 3^N has been included in (5) to take into account the difference between the number of gauge $\{U\}$ and spin $\{S\}$ configurations, since for each spin configuration with N sites we have 3^N identical gauge ones.

A gauge invariant quantity $\langle\delta_{U_{p_i},1} + \delta_{U_{p_i},z}\rangle$ is the order parameter of the model.

The ferromagnetic BEG model on the usual Bethe lattice has been exactly solved in [8, 9], where the λ-line of the second-order phase transition ending in the tricritical point has been found. Thus we can rewrite these solutions in terms of the gauge couplings. For the λ-line we have

$$\exp(\beta_g^\lambda) = 2(b - u_0)(u_0 + 1)^3 u_0^{-1} \tag{6}$$

$$\langle \delta_{U_{p_i,1}} + \delta_{U_{p_i,2}} \rangle_\lambda = \frac{u_0(1 + u_0)}{b + u_0^2} \tag{7}$$

where

$$b = \exp\left(\tfrac{1}{2}(\alpha_{2g} + \beta_{2g})\right) \cosh\left(\tfrac{1}{2}(\alpha_{2g} - \beta_{2g})\right) - 1$$
$$a = b/\left(\exp\left(\tfrac{1}{2}(\alpha_{2g} + \beta_{2g})\right) \sinh\left(\tfrac{1}{2}(\alpha_{2g} - \beta_{2g})\right)\right)$$
$$u_0 = a/(3 - a)\ .$$

The tricritical point is determined by the following expression:

$$\frac{u_0 + 1}{b - u_0} = 2 + \frac{1}{8u_0}\ . \tag{8}$$

Figure 1 shows the phase diagram of the $Z(3)$ gauge model for the value $\beta_{2g}/\alpha_{2g} = \frac{1}{2}$. The λ-curve (full curve) of the second-order phase transition starts at ($\beta_g = -\infty\alpha_{2g} = 2\ln 2$), and finishes at the tricritical point A, which exists for the values $-\infty < \beta_{2g}/\alpha_{2g} < 0.5478$. The tricritical point A cuts the λ-line and separates the second-order phase transition from the first-order one.

We constructed the $Z(3)$ gauge lattice model with double plaquette (window) representation of the action and showed that this model is dual to the spin-1 BEG one. Using the exact solution of the ferromagnetic BEG model on the Bethe lattice we found the line of the second-order phase transition ending in the tricritical point.

The last investigations [15] show that the antiferromagnetic BEG model defined on the Bethe lattice has a rich phase structure, including new ferrimagnetic phase, tetracritical, tricritical and bicritical points, which coincide with results obtained by means of mean-field approximation [16]. Therefore, the same multicritical points exist in the dual gauge model for the following values of the coupling constants $\beta_{2g}/\alpha_{2g} > 1$.

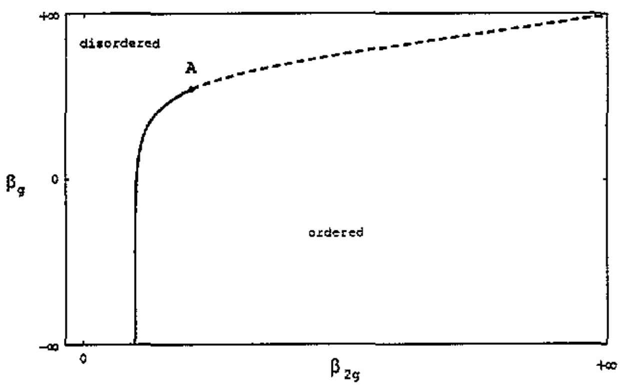

Figure 1. Phase diagram of the $Z(3)$ lattice gauge theory. The λ-curve of the second-order phase transition (full curve) and the line of the first-order phase transition (broken curve) meet together at the tricritical point A. These curves separate the plane of the coupling constants ($\beta_g\alpha_{2g}$) into two regions (ordered and disordered).

We would like to thank R Flume, K Oganessyan and A Akheyan for fruitful discussions. One of us (RRS) wishes to thank V B Priezzhev for the hospitality extended to him at the Bogoliubov Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research where this work has been done. This work was partially supported by German Bundesministerium für Forschung und Technologie under grant no 211-5291 YPI and by the grant INTAS-93-633.

References

[1] Kogut J B 1980 *Phys. Rev.* D **21** 2316
[2] Ananikian N S and Akheyan A Z 1991 *Sov. J. Nucl. Phys.* **53** 1143
Akheyan A Z and Ananikian N S 1992 *J. Phys. A: Math. Gen.* **25** 3111
[3] Griffiths R B 1970 *Phys. Rev. Lett.* **21** 715
Blume M, Emery V J and Griffiths R B 1971 *Phys. Rev.* A **4** 1071
[4] Berker A N and Wortis M 1976 *Phys. Rev.* B **14** 4946
de Alcantara Bonfim O F and Sá Barreto F C 1985 *Phys. Lett.* **109A** 341
[5] Osorio R 1989 *J. Phys. C: Solid State Phys.* **1** 6887
[6] Horiguchi T 1986 *Phys. Lett.* **113A** 425
Wu X N and Wu F Y 1988 *J. Stat. Phys.* **50** 41
[7] Tang K-F 1988 *Phys. Lett.* **133A** 183
[8] Avakian A R, Ananikian N S and Izmailian N S 1990 *Phys. Lett.* **150A** 163
Ananikian N S, Avakian A R and Izmailian N S 1991 *Physica* **172A** 391
[9] Ananikian N S, Izmailian N S and Shcherbakov R R 1994 *JETP Lett.* **59** 71
[10] Ananikian N S and Shcherbakov R R 1994 The interrelation of a $Z(3)$ gauge theory on the flat lattices and a spin-1 BEG model *Proc. VIII Int. Seminar: Quarks-94* (Singapore: World Scientific) at press
[11] Essam J W and Fisher M E 1970 *Rev. Mod. Phys.* **42** 272
[12] Edgar R C 1982 *Nucl. Phys.* B **200** 345
[13] Bhanot G, Drouffe J M, Schiller A and Stamatescu I O 1983 *Phys. Lett.* **125B** 67
[14] Turban L 1984 *J. Phys. A: Math. Gen.* **17** 419
[15] Akheyan A Z, Ananikian N S and Izmailian N S 1994 to be published.
[16] Hoston W and Berker N 1991 *Phys. Rev. Lett.* **67** 1027